Lab 10.5 — Planetary Models

The Checklists

Components Checklist

- ☐ Completed packet
- ☐ Printed & stapled Excel sheet
- ☐ Station is cleaned up for the next group
- ☐ Learning Target Assessment (optional)

Completion Checklist

- ☐ Answered all parts of questions
- ☐ Units included whenever necessary
- ☐ Values with uncertainty were rounded appropriately
- ☐ Graphs have all components
    - Title (DV vs. IV is fine)
    - Axis labels with units
    - Trendline
    - Trendline equation
- ☐ Reality checks
    - Does your math look right to you?
    - Does your Excel output make sense?
        - Do the ranges of numbers on graph axes match the data you think you're graphing on those axes?
        - Does your LINEST output match the trendline equation on your graph?
        - Is your LINEST output much larger or smaller than your expected values?

Specific steps to submit your completed Excel work:

1. Make sure your work is open on the desktop version of Excel (the browser version can't print formulas properly)
2. On the toolbar, click Formulas > Show Formulas.
3. **Move your graphs and adjust the column widths so that things print nicely!**
    - ★ Resize the columns to be as small as possible without cutting off your formulas.
4. Print to the lab printer (Herak 257 Printer).
    - ★ At the bottom of the print menu is a dropdown for scaling. Choose Fit All Columns on One Page.

Staple the formula-view version of your Excel work to the back of your lab.

Score:

Goals

The goal of this week's lab is to get more practice and familiarity with power laws and building mathematical models from real-world data.

By the end of this lab, you should feel more comfortable in doing the following:

1. Use natural logs to make power law data graph linearly.
2. Use graphical analysis to determine values for $\boldsymbol{C}$ and $\boldsymbol{n}$ in a power law.
3. Use linear regression analysis to determine values for $\boldsymbol{C}$ and $\boldsymbol{n}$ in a power law.
4. Use data to determine a power law relationship between two physical quantities.

Part 1: Build an astronomical model from scratch

Background

A. Introduction

Planets closer to the sun have short years; planets farther away have long years. For instance, Mercury's year is about 1/4 of an Earth year, while Neptune's year is more than 100x longer than Earth's!

So there appears to be a relationship between a planet's average separation from the sun, called its **semi-major axis ($\boldsymbol{a}$)**, and the length of its year, called its **period ($\boldsymbol{P}$)**. This relationship is called Kepler's Third Law, and mathematically, it looks like this:

$$P^2 = a^3$$

However, Kepler came up with this law in the early 1600s. We didn't know about Uranus or Neptune yet, let alone about Pluto and the other Kuiper Belt objects. Needless to say, we have much more precise and complete data now. We've been to space! Kepler never went to space! What does he know??

So what we're going to do today is to see whether Kepler's Third Law still works or whether it needs to be updated, using data from NASA on our solar system.

B. Familiarize yourself with your data

Go to **Canvas > Modules > Lab 10.5** and open **10-5_Data.xlsx**. On the first page, called **Part 1: Our Solar System,** you'll see each planet's separation from the sun in AU, and its period in years.

- AU means "astronomical units," and 1 AU is about equal to the Earth's separation from the sun.
- Years is in terms of Earth years.

So we're measuring everything relative to the Earth. But you'll also notice that, while everything is measured relative to the Earth, Earth's values are *about* 1 AU for separation and 1 year for period. That's because our data have grown more precise since we defined the AU unit of measurement, and also because leap years happen.

The other info given here is all about object classification. Dwarf planets are also massive enough to form spherical objects instead of irregular objects, but they may not clear their orbits and they may have orbits that tilt at an angle relative to most objects. Some of these dwarf planets live in the Kuiper Belt, a big ring of small objects — dwarf planets, asteroids, and comets — that starts past Neptune.

Build a model!

Our goal here is to get a mathematical model that describes how period (P) depends on separation (a). We'll assume that this model follows a power law and takes the form $y = Cx^n$.

1. Identify your IV and your DV. Explain your reasoning.

2. Find n by doing the following:
    a. In Processed Data, make two new columns, **ln(a)** and **ln(P),** and fill them with the natural logs of your **a** and **P** data.
    b. **Make a graph of ln(P) vs. ln(a),** complete with labels and a trendline.
    c. **Perform LINEST() on ln(P) vs. ln(a)** in the designated LINEST() area.

3. Now, we'll settle on a value for n by doing a four-line summary.

BE ± Unc	
Range	
Expected value (from Kepler's Third Law)	
Agreement?	

- **If you found agreement**, use the expected value for n for the rest of part 1.
- **If you did not find agreement,** use the value in your range that is the closest to the expected value.

4. Now, we'll find C.
    a. In the Processed Data section, make a column called **a^n**, replacing **n** with the value you just settled on in #3.
    b. **Make a graph of P vs. a^n,** complete with labels and a trendline.
    c. **Perform LINEST() on P vs. a^n** in the designated LINEST() area.

5. How do our C results compare with Kepler's Third Law?

BE $\pm$ Unc	
Range	
Expected value (from Kepler's Third Law)	
Agreement?	

6. So, what do you think? Does Kepler's Third Law still hold up? Does it need to be updated? In your answer, use the results of your analysis and your four-line summaries to justify your claims.

Part 2: How generally applicable is this model?

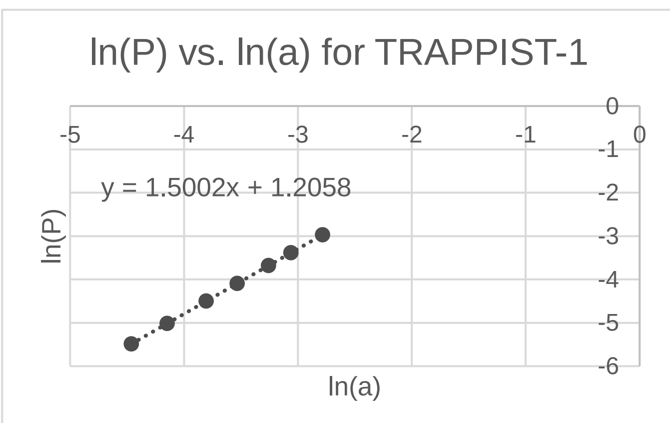
Background

Planets outside of our solar system are called exoplanets, and they're *really cool.* We've found over 5,000 planets outside of our solar system using a variety of detection methods, most of them indirect — meaning we've never seen them, but we can tell that they exist based on periodic effects they have on their host star.

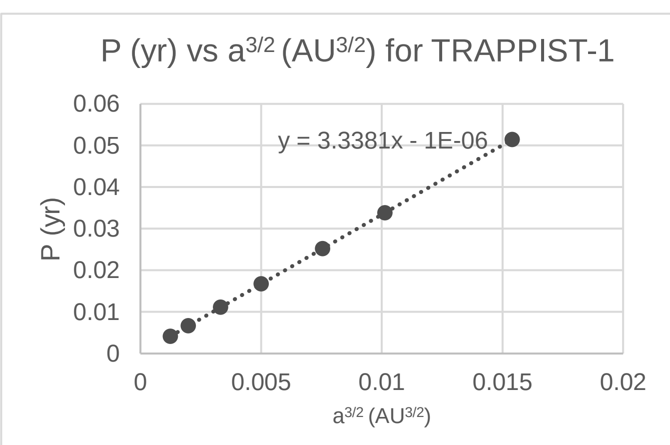
TRAPPIST-1 is a small, relatively cool star almost 40 lightyears away, and it has seven Earth-sized planets orbiting it. They were detected using the transit method, which looks at the periodic dimming of a star's brightness to determine whether something is orbiting it. Given their size. the temperature of their host star, and analyses of some of these planets' atmospheres, we think these planets could have volatile materials on them, which, to astronomers, includes water.

This is *rad as heck*. So I decided to take a look at the TRAPPIST-1 system! Specifically, I wanted to know whether Kepler's Third Law works the same for the TRAPPIST-1 system as it does for our solar system. That is, do solar systems all behave the same way?

Here is a summary of my results.



Raw Data		
Planet	**a (AU)**	**P (yr)**
TRAPPIST-1b	0.01154	0.00413642
TRAPPIST-1c	0.0158	0.0066309
TRAPPIST-1d	0.02227	0.01108616
TRAPPIST-1e	0.02925	0.01670366
TRAPPIST-1f	0.03849	0.02520887
TRAPPIST-1g	0.04683	0.03381915
TRAPPIST-1h	0.06189	0.05139731



Four-Line Summary		
	n (unitless)	**C (yr/AU$^{3/2}$)**
BE ± Unc	1.5001 ± 0.0005	3.3381 ± 0.0009
Range	1.4996 - 1.5007	3.3372 - 3.3390
Expected	1.5	1.001
Agreement?	Yes	***No!***

The TRAPPIST-1 data don't agree with our model! What's the deal with that??

Remember that we are using units *based on the Earth.* I.e., period is in terms of Earth years, and separation is in terms of Earth's distance from the sun. But if we're studying a different solar system — one with a different star and different planets and Earth is not present at all — then using Earth-based units might be a bad idea! What we need is a conversion factor to make our model applicable to any other solar system.

The model that you worked on in Part 1 is the original version of Kepler's Third Law — it's basically the version that Kepler himself worked out. But Kepler was missing some crucial information. It took almost 70 years after Kepler published his law that Newton took a look at Kepler's data and said, "I'm pretty sure there's another, more fundamental relationship here, and I just invented calculus during pandemic quarantine so let me see if that can help me figure this out."

So, after Newton took a stab at it, Kepler's Third Law got an upgrade that made it applicable to any solar system:

$$P = \frac{1}{\sqrt{M+m}} a^{3/2},$$

where $\boldsymbol{M}$ is the mass of the star and $\boldsymbol{m}$ is the mass of a planet, both in solar masses. By measuring our masses in solar masses, we can keep $\boldsymbol{P}$ in years and $\boldsymbol{a}$ in AU. And you can see that the 3/2 exponent that you found in part 1 is there!

So now we've got a model that works for other solar systems, but it would be *very* difficult to work with using the tools we've learned so far, because the value of $\boldsymbol{M + m}$ is going to change depending on which planet's data you're working with.

But maybe there's a simplifying assumption we can make here. Planet masses are much smaller than star masses, so $\boldsymbol{M + m}$ can't be much larger than $\boldsymbol{M}$ alone. So maybe we can rewrite our model to be

$$P = \frac{1}{\sqrt{M}} a^{3/2},$$

because $\boldsymbol{M + m \approx M}$. When looking at our own solar system, by plugging in M = 1 M_{sun} for our daystar, we have exactly the model we got in part 1.

But is this a viable assumption in a system with a much smaller star? In order to find out, we need to use our TRAPPIST-1 data to verify this simplified model.

Test our revised model

Scroll down to **Part 2: TRAPPIST-1**. Because we're testing a developed model, we don't have to do a bunch of logarithms; we can cut to the chase and graph **P** vs **$a^{3/2}$**. By doing this, we are essentially linearizing by substitution as we did in labs 8 and 9, so we can also predict what the y-intercept should be.

7. Calculate the expected value for the slope, **with units,** of the best fit line of a **P** vs $\mathbf{a^{3/2}}$ graph.
   - M =0.09 M_{sun} (This is the mass of TRAPPIST-1, the host star of the system.)
   - ★ You can show your work to tell me what you're plugging in and then use a free cell in Excel to do the math.

8. What is the expected value, **with units,** for the y-intercept?

9. Now, we'll find our experimental slope and y-intercept.
   a. In the Processed Data section, make a column called **a^3/2** and calculate $\boldsymbol{a^{3/2}}$ for each of your **a** values.
   b. **Make a graph of P** vs $\mathbf{a^{3/2}}$**,** complete with labels and a trendline.
   c. **Perform LINEST() on P** vs $\mathbf{a^{3/2}}$ in the designated LINEST() area.

10. We have our expectations, and we have our experimental values, so now we can determine agreement! Fill in the table below, making sure to round for sigfigs and include units.

	Slope	**y-intercept**
BE ± Unc		
Range		
Expected value (from #7 & #8)		
Agreement?		

11. Based on the results of your analysis, do you believe our choice to replace $M+m$ with just M alone was reasonable? **Explain your reasoning.** (And note that the amount of space available here is not a suggestion for how much you should write; it's just how the document layout ended up.)