PLL 10: Building Models from Scratch

First: Announcements, Debriefs, Etc.

This Week's System(s)	This Week's Question(s)

Where do mathematical models come from?

Brainstorm: How can we use the skills we've learned so far to study a system for which a mathematical model does not exist?

Power Laws

We're going to assume that the systems we're studying this week and next week are ***power law*** functions.

The general form of a power law is

$$y = Cx^n$$

(And if I were you, I'd label what each of those variables mean.)

A note on n: n can be any value: if $n = 1$, you get a straight line; if $n = 2$, you get a squared function; if $n = 1/2$, you get a square root ... etc. So "power law functions" is actually a big umbrella that lots of different types of equations full under — making it a decent starting assumption when looking at raw data for a system you know almost nothing about.

Sample Problem: Map each of the following power law models to the general form of a power law — that is, in each equation, identify ...

- What plays the role of y?
- What plays the role of x?
- What's the value of C?
- What's the value of n?

1. Model for the kinetic energy of a moving object:
   $$KE = \frac{930}{2} \text{ kg } \cdot v^2$$

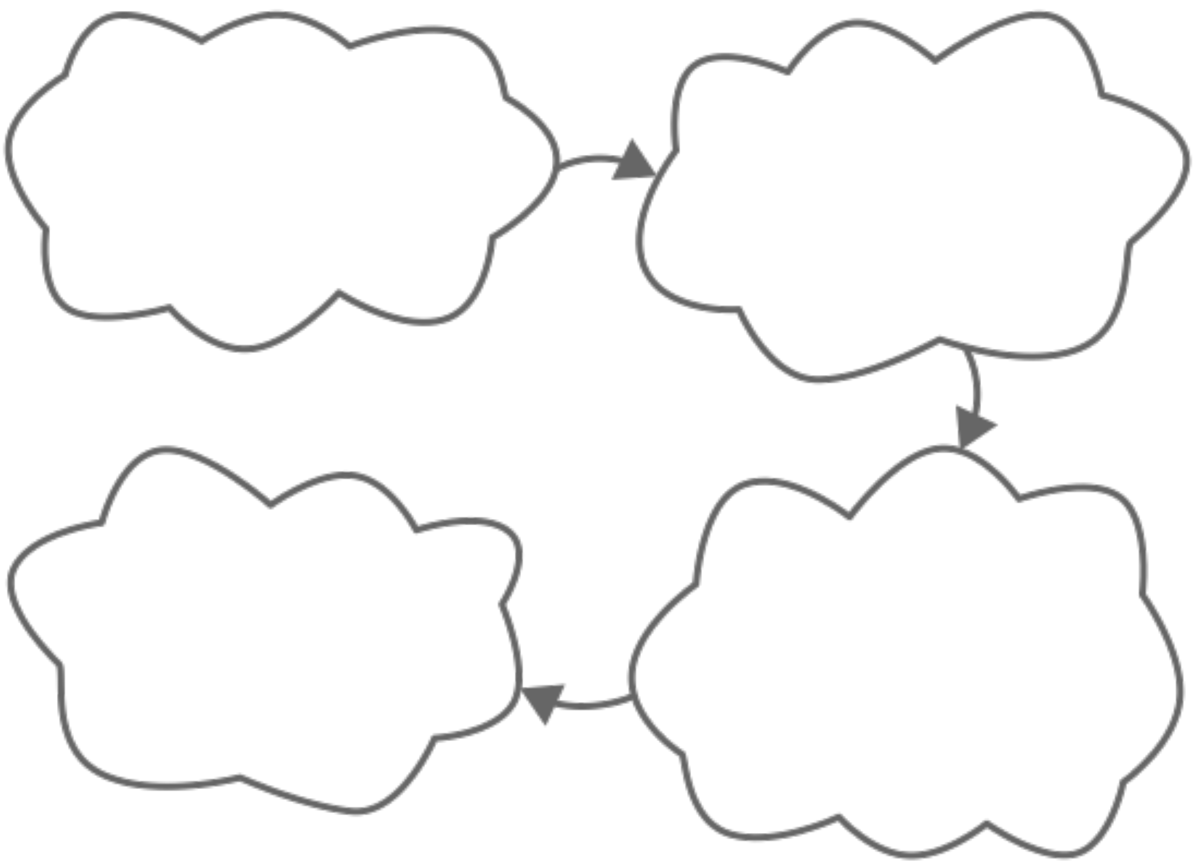
2. Model for the force that can be exerted by an accelerating object:
   $$F = 13 \text{ kg } \cdot a$$

3. Model for the relationship between a pendulum's period and its length:
   $T = \frac{2\pi}{\sqrt{g}} L^{\frac{1}{2}}$ (and remember that $g = 9.81$ m/s^2)

Q4U: Looking at those models, what's something that C values have that n values don't have? What might that mean about what C and n each tell us about the system being modeled by a particular power law equation?

Using Power Laws to Build New Models

General Overview of Process



Step One: Gathering and taking logs of your data

The Process

1. ______________________________

2. ______________________________

3. ______________________________

Why it works: Logarithms and exponents go hand in hand!

Natural logs (aka $\ln$)

- Logarithms in general: $\log_b(a) = c$
  - $b \rightarrow$ ______________________________
  - a and $c \rightarrow$ ______________________________
- The natural logarithm, $\ln$ uses base e!
  - Euler's constant $e = \boxed{}$

$$\ln \_\_ = \_\_ \qquad e^{-} = \_\_\_$$

The logarithm rules:

1. ______________________________

2. ______________________________

3. ______________________________

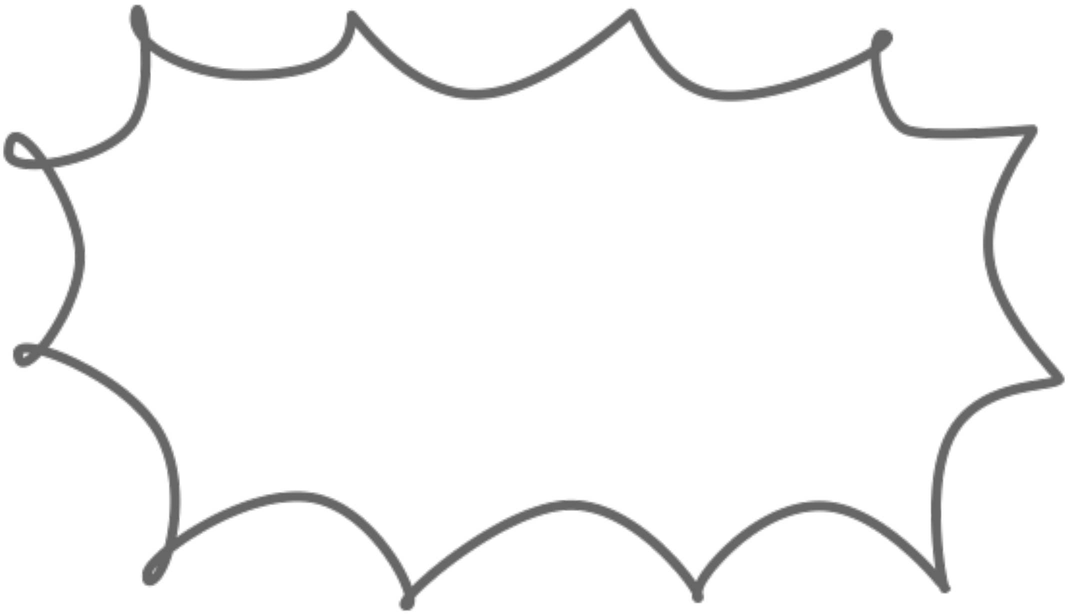
Q4U: Why do we need to take the log of both sides of the equation?

Starting with the general form of a power law,

$$y = Cx^n$$

1. take the natural log of both sides of the equation.
2. use the relevant logarithm rules to
   (a) separate C from x^n
   (b) separate n from x.

(Note that you won't have to use all three log rules!)



Step Two: Use the ln-ln plot to find n

The Process

1. ____

2. ____

3. ____

Why it works: We can use our growing understanding of the system to make informed choices.

What do models tell us?	What do data tell us?

What this means for us:

Think-Pair-Share: Lets say I'm interested in how the saplings of a particular species of tree grow, and I want to look specifically at how the sapling's total mass depends on its width. I measure a bunch of trees' diameters (d) and masses (m), log my data, graph them and do LINEST, and get the following results.

	Slope	Y-Int
BE	1.80	2.1
SE	0.05	0.1
AE	0.09	0.2
Min	1.71	1.9
Max	1.89	2.3

I know that in some closely related species of trees, the cross-sectional area determines how much water and nutrients can be transported through the trunk for growth. Area is a 2-dimensional quantity, so I'd expect $n = 2$. Given my data, what value for n should I choose, and why?

Step Three: Use use a y vs. x^n plot to find C

The Process

1. ______________________________

2. ______________________________

3. ______________________________

Why it works: This part is just the same linearization process from labs 8 and 9!

Step Four: Write your mathematical model!

The Process

Starting with the general form of a power law

$$y = Cx^n$$

1. ______________________________

2. ______________________________

3. ______________________________

WE DID IT! WE HAVE A MODEL!